Exercise 55

Find $R'(0)$, where

$$R(x) = \frac{x - 3x^3 + 5x^5}{1 + 3x^3 + 6x^6 + 9x^9}$$

Hint: Instead of finding $R'(x)$ first, let $f(x)$ be the numerator and $g(x)$ the denominator of $R(x)$ and compute $R'(0)$ from $f(0)$, $f'(0)$, $g(0)$, and $g'(0)$.

Solution

Following the hint, set

$$f(x) = x - 3x^3 + 5x^5 \qquad \rightarrow \qquad f(0) = 0$$

$$g(x) = 1 + 3x^3 + 6x^6 + 9x^9 \qquad \rightarrow \qquad g(0) = 1.$$

Then

$$R(x) = \frac{f(x)}{g(x)}.$$

Take the derivative using the quotient rule.

$$R'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

Set $x = 0$.

$$R'(0) = \frac{f'(0)g(0) - g'(0)f(0)}{[g(0)]^2}$$

Take the derivative of $f(x)$ and $g(x)$.

$$f'(x) = 1 - 9x^2 + 25x^4 \qquad \rightarrow \qquad f'(0) = 1$$

$$g'(x) = 9x^2 + 36x^5 + 81x^8 \qquad \rightarrow \qquad g'(0) = 0$$

Therefore,

$$R'(0) = \frac{(1)(1) - (0)(0)}{(1)^2} = 1.$$